



This is just ONE way to derive this relationship.
There are two area formulas that come into play:

$$Area = \frac{1}{2}(AB)(AC)\sin(x)$$

which I refer to as the SAS Area formula, and

$$Area = rs$$

where r is the inradius and s is the semi perimeter.

Combining these two area formulas, we solve for the radius r in terms of x and one of the legs (which we hold constant).

$$rs = \frac{1}{2}(AB)(AC)\sin(x)$$

$$rp = (AB)(AC)\sin(x)$$

$$rp = (AB)^2\sin(x)$$

$$r(AB + AC + BC) = (AB)^2\sin(x)$$

$$r = \frac{(AB)^2\sin(x)}{AB + AC + BC}$$

$$r = \frac{(AB)^2\sin(x)}{2AB + BC}$$

$$r = \frac{(AB)^2\sin(x)}{2AB + \sqrt{AB^2 + AC^2 - 2(AB)(AC)\cos(x)}}$$

$$r = \frac{(AB)^2\sin(x)}{2AB + \sqrt{AB^2 + AB^2 - 2(AB)(AB)\cos(x)}}$$

$$r = \frac{(AB)^2\sin(x)}{2AB + \sqrt{2AB^2 - 2(AB)^2\cos(x)}}$$

Multiplying both sides by 2 gives leads to $2s$ on the left side, which is the perimeter p .

$$AB = AC$$

Using the Law of Cosines to express BC in terms of x and the legs of the triangle.

This allows us to express the area in terms of x and the legs (which are held constant).

$$Area = \pi r^2$$

$$Area = \pi \left(\frac{(AB)^2 \sin(x)}{2AB + \sqrt{2AB^2 - 2(AB)^2 \cos(x)}} \right)^2$$

This formula for the area is actually different from the formula in the applet.